from the piston valve to accelerate the formation of the shock-wave front. In previously suggested valve designs, values of L/D from 20 to 40 for $M_s = 1.61$ were quoted.[2] It is apparent that the present valve system exhibits good performance even with a heavy piston valve while having a relatively simple structure.

Obtained shock-wave Mach numbers are almost the same values as the analytical ones. In previous shock-wave valve designs,[2] the obtained shock-wave Mach number was between 60 and 90% of the analytical value.

Conclusions

To overcome the defects of diaphragm-type shock tubes, a double-piston-type shock-wave valve has been designed and constructed. The results obtained from the proposed new valve are summarized as follows.

1) Even though this valve has a simple design and the piston is comparatively heavy, smooth postshock pressures were obtained for incident shock-wave Mach numbers of $M_s = 1.2$ and higher.

2) Shock-wave formation length is about 65 calculated shock-tube diameters for $M_s = 1.2$; this is comparable to what is obtained with other types of shock-wave valves.

3) The Mach number obtained with this valve is very close to analytically predicted values, and the pressure loss at the valve is small.

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Role of Matrix in Viscoplastic Behavior of Thermoplastic Composites at Elevated Temperature

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Introduction

THE ductility of thermoplastic resin improved the mechanical and environmental characteristics of fiber composites but induced the complicated viscoplastic behavior of fiber composites. Viscoplastic behavior is intrinsically nonlinear and the anisotropy of composites increases the difficulties of describing the viscoplastic behavior of thermoplastic composites. Therefore, to use the thermoplastic composites properly, the viscoplastic behavior of thermoplastic composites must be studied more thoroughly. And also the role of a matrix in viscoplastic behavior of thermoplastic composites needs to be investigated since the viscoplasticity of thermoplastic composites is from the ductility of thermoplastic resin.

Many studies[1-3] were reported for describing the nonlinear behavior of fiber composites. But the role of a matrix in viscoplastic behavior of thermoplastic composites was not investigated sufficiently in previous studies. In this study, to investigate the role of a matrix in viscoplastic behavior of thermoplastic composites, the concept, what we call "unmixing-mixing," is suggested. This concept makes us use not viscoplastic rule of anisotopic material (composite) but viscoplastic rule of isotropic material (matrix). Since many physical observations show that fibers deform linear elastically and matrices exhibit viscoplastic behavior, it can be said that the viscoplastic behavior of a matrix is the main factor in viscoplastic behavior of overall thermoplastic composites. Moreover, fiber composite is a mixture of isotropic materials (fiber and matrix). Thus, it is meaningful and plausible that the viscoplastic behavior of thermoplastic composites be predicted by using a viscoplastic rule of isotropic material (matrix).

Concept of "Unmixing and Mixing"

The following assumptions are used in this study.

1) Unidirectional composites considered in this study are in the plane stress state.

2) Strains of matrix and fiber are the same in fiber direction.

3) Total strain rate can be decomposed into elastic and plastic components.

4) Fibers and matrices exhibit linear elastic and viscoplastic behaviors, respectively.

To describe the elastic properties of composites in terms of constituent properties and to satisfy the above assumptions, the rule of mixture[4] in micromechanics is used. This rule of mixture considers the effect that the Poisson's ratio difference of fiber and matrix induces force in fiber direction under transverse loading conditions. The rule of mixture is as follows:

$$E_1 = E_f V_f + E_m V_m \tag{1a}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \tag{1b}$$

$$1/E_2 = V_f/E_f + V_m/E_m - V_f V_m (\nu_f^2 E_m/E_f + \nu_m^2 E_f/E_m - 2\nu_f \nu_m)/(E_f V_f + E_m V_m) \tag{1c}$$

$$1/G_{12} = V_f/G_f + V_m/G_m \tag{1d}$$

where V denotes the volume fraction. Subscript f and m denote fiber and matrix, respectively, and subscript 1 and 2 denote the material principal axes (fiber direction and transverse direction). The effect of Poisson's ratio difference is presented in the third right-hand side term for E_2.

From the assumptions 2, 3, and 4 with kinematic observations, strain rates in (1,2) coordinate can be expressed in terms of elastic and plastic components.

$$\dot\epsilon_{11} = \dot\epsilon^e_{f_{11}} = \dot\epsilon^e_{m_{11}} + \dot\epsilon^p_{m_{11}} \tag{2a}$$

$$\dot\epsilon_{22} = V_f \dot\epsilon^e_{f_{22}} + V_m(\dot\epsilon^e_{m_{22}} + \dot\epsilon^p_{m_{22}}) = \dot\epsilon^e_{22} + V_m \dot\epsilon^p_{m_{22}} \tag{2b}$$

$$\dot\epsilon_{12} = V_f \dot\epsilon^e_{f_{12}} + V_m(\dot\epsilon^e_{m_{12}} + \dot\epsilon^p_{m_{12}}) = \dot\epsilon^e_{12} + V_m \dot\epsilon^p_{m_{12}} \tag{2c}$$

where superscript e and p denote elastic component and plastic component, respectively. The 1-direction strains of matrix and fiber are obtained from the elastic stress-strain relations with the assumptions 1, 3, and 4.

$$\dot\epsilon_{f_{11}} = \dot\epsilon^e_{f_{11}} = \dot\sigma_{f_{11}}/E_f - (\nu_f/E_f)\,\dot\sigma_{f_{22}} \tag{3a}$$

$$\dot\epsilon_{m_{11}} = \dot\epsilon^e_{m_{11}} + \dot\epsilon^p_{m_{11}} = \dot\sigma_{m_{11}}/E_m - (\nu_m/E_m)\,\dot\sigma_{m_{22}} + \dot\epsilon^p_{m_{11}} \tag{3b}$$

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With the force equilibrium equations in (1, 2) coordinate such that $\dot\sigma_{11} = V_m \dot\sigma_{m_{11}} + V_f \dot\sigma_{f_{11}}$, $\dot\sigma_{22} = \dot\sigma_{m_{22}} = \dot\sigma_{f_{22}}$, and $\dot\sigma_{12} = \dot\sigma_{m_{12}} = \dot\sigma_{f_{12}}$, applications of assumption 1 to Eqs. (3a) and (3b) give the following expressions:

$$\dot\sigma_{m_{11}} = (E_m/E_1)\dot\sigma_{11} + [(\nu_m E_f - \nu_f E_m)/E_1] V_f \dot\sigma_{22} - (E_m E_f V_f / E_1)\dot\epsilon^p_{m_{11}} \tag{4a}$$

$$\dot\sigma_{f_{11}} = (E_f/E_1)\dot\sigma_{11} + [(\nu_f E_m - \nu_m E_f)/E_1] V_m \dot\sigma_{22} + (E_m E_f V_m / E_1)\dot\epsilon^p_{m_{11}} \tag{4b}$$

These show that fiber takes much more force in the plastic region than in the elastic region due to the plastic deformation of matrix.

Since $\dot\sigma_{22}$ is equal to $\dot\sigma_{m_{22}}$, substitution of Eqs. (4) into Eqs. (3) yields the 1-direction fiber composite strain rate in terms of fiber composite overall stresses and matrix plastic strain.

$$\dot\epsilon_{11} = \dot\sigma_{11}/E_1 - (\nu_{12}/E_1)\dot\sigma_{22} + (E_m V_m/E_1)\dot\epsilon^p_{m_{11}} \tag{5}$$

Substitutions of the overall fiber composite elastic stress-strain relations into $\dot\epsilon_{22}$, $\dot\epsilon_{12}$ in Eq. (2) produce

$$\dot\epsilon_{22} = -(\nu_{12}/E_1)\,\dot\sigma_{11} + (1/E_2)\dot\sigma_{22} + V_m \dot\epsilon^p_{m_{22}} \tag{6a}$$

$$\dot\epsilon_{12} = (1/2G_{12})\dot\sigma_{12} + V_m \dot\epsilon^p_{m_{12}} \tag{6b}$$

Finally, from Eqs. (5) and (6), strain rates in (1, 2) coordinate can be written in tensor form as follows.

$$\dot{\boldsymbol\epsilon} = \boldsymbol A \dot{\boldsymbol\sigma} + \boldsymbol B \dot{\boldsymbol\epsilon}^p_m \tag{7}$$

The first and second right-hand side terms in the above equation are the elastic and the plastic parts, respectively. It is a reasonable result that the viscoplastic behavior of overall fiber composites depends on the matrix plastic strain rates, elastic properties, and the volume fraction.

As obtained before [force equilibrium in (1, 2) coordinate, (4a)], stress rates undertaken by a matrix have the following tensor form:

$$\dot{\boldsymbol\sigma}_m = \boldsymbol C \dot{\boldsymbol\sigma} + \boldsymbol D \dot{\boldsymbol\epsilon}^p_m \tag{8}$$

As Eqs. (7) and (8) mean that $\dot\epsilon = fn(\dot\sigma, \dot\epsilon^p_m)$ and $\dot\sigma_m = fn(\dot\sigma, \dot\epsilon^p_m)$, so if we have a matrix viscoplastic constitutive model $\dot\epsilon^p_m = fn(\sigma_m)$, then we can relate $\dot\sigma$ to $\dot\epsilon$.

Viscoplastic Analysis with Modified Bodner and Partom Theory

To apply the "unmixing-mixing" concept with the viscoplastic effect, a modified Bodner and Partom flow rule[5,6] by Kim and Oden which does not require yield criteria or loading and unloading conditions was adopted for matrix viscoplastic constitutive model. By using the generalized potentials[7] based on the theory of materials of type N, a modified Bodner and Partom flow rule was obtained.

The resulting modified Bodner and Partom flow rule is the following form:

$$\sigma_m = \frac{\rho\lambda}{\rho_0}\,\mathrm{tr}(\epsilon^e_m)\mathbf{1} + (\rho/\rho_0)2\mu\epsilon^e_m \tag{9}$$

$$h = (\rho/\rho_0)[h_1 + (h_0 - h_1)\exp(-m\Lambda)] \tag{10}$$

$$\dot\epsilon^p_m = (D_0/2h\sqrt{J_{2D}})\exp[-(Bh^{2n}/J_{2D}{}^n)]\,S_m \tag{11}$$

$$\dot\Lambda = (1/h)S_m : \dot\epsilon^p_m \tag{12}$$

Here

$$S_m = \sigma_m - (1/3)\,\mathrm{tr}(\sigma_m)\mathbf{1} \tag{13}$$

$$J_{2D} = (1/2)\,\mathrm{tr}(S_m)^2 \tag{14}$$

and Λ is an internal state variable (since these potentials will characterize an isotropic hardening effect, here the internal state variable is scalar-valued), h is a hardness variable which is conjugate to the internal variable, λ and μ are Lame constants, and D_0, h_0, h_1, m, and n are material constants. B is equal to $3^{-n}(n+1)/n$.

With the incompressibility condition commonly used in plasticity, system of Eqs. (7–14) can be solved for $\dot\sigma$ with given $\dot\epsilon$ (where σ, σ_m, S_m, $\dot\epsilon^p_m$, ϵ^e_m, J_{2D}, h, and Λ are unknowns). Direct iteration method, bisection method, and Newton-Raphson method were utilized to solve the system of nonlinear equations. Elastic properties of thermoplastic composites at 250°F are listed as follows: $E_1 = 128.2$ GPa, $E_2 = 8.3$ GPa, $G_{12} = 4.9$ GPa, and $\nu_{12} = 0.32$. The volume fraction and elastic properties of constituents used in this study are as follows: $E_f = 211.8$ GPa, $\nu_f = 0.27$, $V_f = 0.6$, $E_m = 2.8$ GPa, $\nu_m = 0.395$, and $V_m = 0.4$. The measured data of AS4/PEEK thermoplastic composites are from Ref. 2. The material constants for the viscoplastic law are determined by curve fitting to the experimental data[2] at the slowest and the fastest rates of 15-deg off-axis specimen. The determined constants are as follows: $D_0 = 831.6 \times 10^6$ MPa/s, $h_0 = 155.6$ MPa, $h_1 = 304$ MPa, $m = 250$, and $n = 1$.

By the "unmixing-mixing" scheme with the modified Bodner-Partom flow rule, stress-strain curves of 15-, 30-, and 45-deg off-axis thermoplastic composites for 10^{-3}/s, 10^{-6}/s strain rates at 250°F are predicted. The simulated results are compared with the experimental results as shown in Fig. 1. Since the viscoplastic material constants are obtained from 15-deg off-axis specimen, the simulated results for 30- and 45-deg cases show some discrepancy to the experimental data. To show the viscoplastic effect clearly, the behaviors under rapid change of strain rates are also simulated and the results are shown in Figs. 2 and 3. Figure 2 shows that the immediate responses just after the higher strain rates in the predicted curves of thermoplastic composites are the same as elastic loading processes and jump into the cases of the higher strain rate. On the other hand, Fig. 3 shows that stress-strain curves lead to the unloading processes which are completely elastic and then to the cases of the lower strain rate. The comparisons of the predicted and the measured data show good agreements.

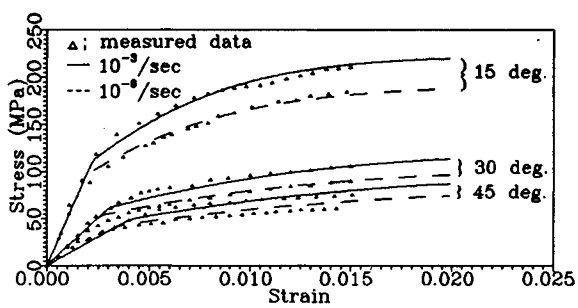

Fig. 1 Measured and predicted stress-strain curves of off-axis specimens for different strain rates at 250°F.

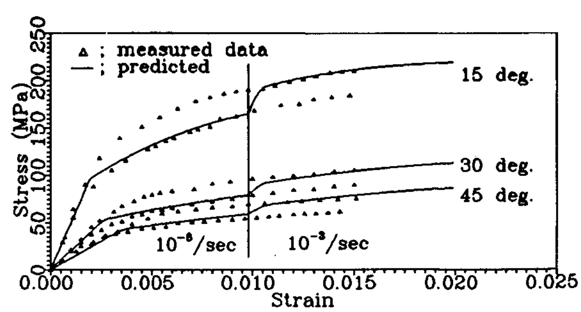

Fig. 2 Measured and predicted stress-strain curves of off-axis specimens subjected to a rapid change (increase) in strain rate at 250°F.

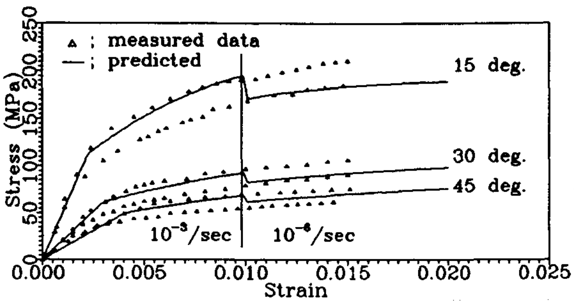

Fig. 3 Measured and predicted stress-strain curves of off-axis specimens subjected to a rapid change (decrease) in strain rate at 250°F.

Conclusions

The role of matrix in viscoplastic behavior of thermoplastic composites is identified in this study. On the basis of the fact that thermoplastic composite is a mixture of isotropic materials (fiber and matrix), the viscoplastic behaviors of thermoplastic composites with various off-axis angle are simulated by using the "unmixing-mixing" concept and a modified Bodner-Partom flow rule as a matrix viscoplastic constitutive model. As a consequence, this work gives us the simplicity in handling of the elastic anisotropy of the thermoplastic composites and makes it possible that the viscoplastic behavior of thermoplastic composites with any off-axis angle can be predicted by the viscoplastic properties of the matrix only. The concept of this study can be extended to the analysis of the laminated structures by solving boundary-value problems.

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Substructure Decomposition Method for the Control Design of Large Flexible Structures

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Introduction

Because of the increasing demands of high structural performance requirements, the control of large flexible structures has attracted a considerable amount of research in recent years. There are two basic steps involved in the control of a flexible structure using discrete methods. In the first step, the structure is modeled, usually by the finite element method (FEM) that, for a large flexible structure, results in a large number of ordinary differential equations. These dynamic equations of motion (EOM) have to be used in the second step to determine a control law appropriate for the structure. In many cases, it is not practical or feasible to consider the full dynamic model of the structure for the global controller design. The decentralized control design is very attractive in the active control of large flexible structures, since it permits the design of controllers at component level and avoids many complications that may otherwise be present in the controller design for the whole structure. Recently, an approach called the controlled component synthesis was developed by Young[1] in which an interlocking control concept was used to minimize the motion of the nodes adjacent to the boundaries of the substructures.

In this work, a general decentralized control approach is presented for large flexible structures. The structure is decomposed into substructures for which the linear quadratic regulator (LQR) theory is used to determine the decentralized controllers at substructure level (subcontrollers). The reaction forces at the substructure boundaries were balanced by the control forces generated within the substructures. The subcontroller of each substructure is then used to assemble the controller for the whole structure. The computational effort required in the approach is substantially less compared with the complete structural model.

Substructure Decomposition

It is assumed that the flexible structure is decomposed into r substructures, where the kth substructure has neighboring s substructures. Note that in the subsequent discussions the subscript k refers to the kth substructure. Let $\{I\}_k$ denote the set of internal degrees of freedom (DOF) of the kth substructure, $\{B_1\}_k$ represent the set of boundary DOF between the kth and $(k+1)$th substructures, and so on. The EOM of the kth substructure can be represented as

$$M_k\ddot{x}_k + C_k\dot{x}_k + K_kx_k = D_ku_k \tag{1}$$

where x_k and u_k denote the vectors of displacement and control input, respectively; and M_k, C_k, K_k, and D_k denote the mass, damping, stiffness, and input weighting matrices in the configuration space, respectively. The matrices M_k, C_k, K_k, and D_k have to be summed properly to preserve the displacement compatibility of the whole structure.

The EOM of the kth substructure, Eq. (1), can be rearranged by grouping together the internal and boundary DOF of the substructure. The internal and boundary DOF of a substructure will vary depending on the surrounding substructure considered. Considering the kth substructure and its neighboring $(k+1)$th substructure, the set of internal DOF is given by $\{A_k\} = (\{I_k\}, \{B_2\}, \ldots, \{B_s\})_k$ and the set of boundary DOF by $\{B_k\} = \{B_1\}_k$. Hence the partitioned EOM for the kth substructure can be stated as

$$\begin{bmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{bmatrix}_k \begin{Bmatrix} \ddot{x}_A \\ \ddot{x}_B \end{Bmatrix}_k + \begin{bmatrix} C_{AA} & C_{AB} \\ C_{BA} & C_{BB} \end{bmatrix}_k \begin{Bmatrix} \dot{x}_A \\ \dot{x}_B \end{Bmatrix}_k + \begin{bmatrix} K_{AA} & K_{AB} \\ K_{BA} & K_{BB} \end{bmatrix}_k \begin{Bmatrix} x_A \\ x_B \end{Bmatrix}_k = \begin{bmatrix} D_{AA} & D_{AB} \\ D_{BA} & D_{BB} \end{bmatrix}_k \begin{Bmatrix} u_A \\ u_B \end{Bmatrix}_k \tag{2}$$

By using the Guyan static condensation method, the following relation can be written:

$$\begin{Bmatrix} x_A \\ x_B \end{Bmatrix}_k = \begin{bmatrix} -K_{AA}^{-1}K_{AB} \\ I_B \end{bmatrix}_k x_{Bk} = T_kx_{Bk} \tag{3}$$

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